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USE OF R-MATRIX METHODS FOR LIGHT ELEMENT EVALUATIONS

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ABSTRACT

Some general aspects of parameterizing nuclear reaction data with a unitary, multichannel theory are discussed. The special case of R-matrix theory is considered, where the explicit separation of long- and short-range forces and the natural occurrence of energy pole terms afford a number of advantages in describing data for light-element reactions. Examples are given for both neutron- and charged-particle-induced reactions which illustrate multichannel R-matrix techniques, including the use of charge symmetry to relate data for mirror systems. The limitations of conventional R-matrix methods are discussed briefly.

INTRODUCTION

Nuclear data evaluation often involves the parametric representation of experimental measurements to produce smooth functions of incident energy and scattering angle. To build in even the simplest of the conservation laws for nuclear interactions, e.g., flux, total angular momentum, and parity, one needs to parameterize the unitary collision matrix, U. [1] The most familiar of these is a direct parameterization of U in terms of phase shifts, absorptions, and mixing angles. However, the unitary realizations of U in terms of these parameters becomes increasingly cumbersome as the number of coupled states exceeds two. Furthermore, the energy dependence of these parameters, which comes in great part (especially at low energies) from the known long-ranged parts of the interactions, remains unspecified.

These difficulties with the direct parameterization of the collision matrix can be circumvented by using R-matrix theory. [2] Following a simplified illustration of multi-reaction data analyses using a unitary description, we shall introduce the parameters of R-matrix theory in a brief summary of the formalism.

Unitary Description of Nuclear Reactions

At sufficiently low energies, a system of interacting nuclear particles eventually separates into pairs of clustered fragments which mutually interact only through Coulomb forces (if present). In regions of configuration space where this separation holds (the "internal region"), the radial wavefunction describing the relative motion of any possible two-body arrangement channel c can be expressed as

$$u_c(r_c) = I_c(r_c) - \sum_{c'} O_{c'}(r_c)\, U_{c'c} \quad , \tag{1}$$

where the collision matrix elements $U_{c'c}$ are amplitudes of outgoing spherical waves $O_{c'}$ in each of the channels c' for incoming spherical waves I_c normalized to unit flux incident in channel c. Conservation of the (unit) incident flux in all the outgoing channels implies unitary of the collision matrix,

$$UU^\dagger = U^\dagger U = 1 \quad . \tag{2}$$

The results of any scattering measurement done on the system can be expressed as bilinear combinations of the elements of U. In particular, the differential, integrated, and total cross sections for uncharged incident particles are given by

$$\frac{d\sigma_{\alpha'\alpha}}{d\Omega_{\alpha'}} = F_\alpha \sum_L B_L P_L(\cos\theta_{\alpha'}) \quad , \tag{3}$$

with

$$B_L = \frac{1}{4} \sum_{s's} (-1)^{s'-s} \sum_{J_1 J_2 \ell_1' \ell_2' \ell_1 \ell_2} Z(\ell_1' J_1 \ell_2' J_2, s'L)$$

$$\bar{Z}(\ell_1 J_1 \ell_2 J_2, sL)\,\mathrm{Re}(U_{c_1'c_1} - \delta_{c_1'c_1})(U_{c_2'c_2} - \delta_{c_2'c_2})^* \quad ,$$

$$\sigma_{\alpha'\alpha} = \pi F_\alpha \sum_{Js's\ell'\ell} (2J+1)\left|U_{c'c} - \delta_{c'c}\right|^2 \quad , \tag{4}$$

$$\sigma_{T,\alpha} = 2\pi F_\alpha \sum_{s\ell} 1 - \mathrm{Re}(U_{cc}) \quad , \tag{5}$$

where the factor $F_\alpha = [k_\alpha^2 (2s_{1\alpha}+1)(2s_{2\alpha}+1)]^{-1}$ is defined in terms of the incident center-of-mass wave numbers k_α and spins $s_{1\alpha}$ and $s_{2\alpha}$. The channel label c contains an arrangement index α and the quantum numbers s, ℓ, and J, for spin, orbital angular momentum, and total angular momentum, respectively. The coefficients $\overline{Z}$ are related to those of Blatt and Biedenharn, as defined in Ref. [3].

The unitary conditions (2) satisfied by the collision matrix impose strong constraints on data for different reactions calculated from relations like Eqs. (3)-(5) for the cross sections. To illustrate this, we consider the simple case of only two coupled states, each of which belongs to a different arrangement α. The 2 x 2 collision matrix is conveniently parameterized in terms of Stapp's "nuclear-bar" parameters [4] by

$$U = \begin{bmatrix} \eta\, e^{2i\delta_1} & i\sqrt{1-\eta^2}\, e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}\, e^{i(\delta_1+\delta_2)} & \eta\, e^{2i\delta_2} \end{bmatrix} . \qquad (6)$$

In addition to being unitary, the matrix is symmetric as is required by time-reversal invariance. [1]

We see the unitary and time-reversal conditions allow a parameterization of U in terms of only three real parameters, δ_1, δ_2, and η, which are, in principle, completely determined by analyzing (1,1) and (2,2) scattering data. Thus, in this simple case, data for the (1,2) reaction are <u>completely redundant</u> with (1,1) and (2,2) scattering data. Of course, in most cases of interest, other states must be taken into account that dilute these simple results, but the tendency of unitarity to relate data for different reactions remains, particularly near resonances where the dominance of a few states more closely approximates the simple situation described above.

The ability to include redundant data from different sources has clear statistical advantages in the determination of parameters by fitting experimental measurements. The parameters are better defined simply because their over determination is increased, and they are influenced less by systematic errors in the measurements, assuming these errors occur in random, uncorrelated ways among data for different reactions. Including measurements of observables other than cross sections (polarizations, etc.) has much the same effect, and in addition, since they depend on different bilinear combinations of the collision matrix elements, tends to eliminate multiple solutions for the parameter values, a well-known problem which plagues phase-shift analyses of cross sections.

From the considerations above, it is clear that evaluation purposes are well served by a parameterization of nuclear reactions having a simple multichannel generalization which, at a minimum, builds in the unitarity and symmetry of the collision matrix. R-matrix theory provides such a parameterization which,

moreover, explicitly contains the dependence of the collision matrix on the known, long-ranged forces. A brief, formal summary of this theory is given in the following section.

R-Matrix Formalism

We outline here, for the sake of completeness, dicussion which has appeared in previous contributions [5], [6] to cross section and evaluation meetings, and which is equivalent to that found in the literature. [2], [3], [7] R-matrix theory presumes there exists a set of finite relative coordinates, called "channel radii," beyond which short-ranged forces vanish, and the wavefunction has the form given in Eq. (1). These channel radii (a_c) define a "channel surface," inside of which (i.e., the "internal" region) the wavefunction can be expressed as a formal solution of the Schroedinger equation

$$\Psi = (H-E+\ )^{-1}\ \ , \tag{7}$$

with the addition of a "boundary condition" operator

$$= \sum_c |c)(c|\ \left(-\frac{\partial}{\partial r_c}\ r_c - B_c\right) \quad , \tag{8}$$

which projects onto the channel surface and makes the internal hamiltonian operator H hermitian. [7] The "channel surface" functions $|c)$ in Eq. (8) are defined in terms of channel spin-angle eigenfunctions of total angular momentum and parity, $Y_c(r_c)$ and the channel reduced masses, m_c, by

$$|c) = \frac{2}{2m_c a_c}^{1/2}\ \frac{\delta(r_c-a_c)}{r_c}\ Y_c(r_c) \quad , \tag{9}$$

and B_c are the real, energy-independent boundary condition numbers which characterize the theory of Wigner and Eisenbud. [2]

Using Eq. (8) in the projection of Eq. (7) on the channel surface gives

$$(c'|\Psi) = \sum_c (c'|G|c)(c|\ -\frac{\partial}{\partial r_c}\ r_c - B_c|\Psi) \quad , \tag{10}$$

where the Green's function operator

$$G = (H+\ \ -E)^{-1} \tag{11}$$

is hermitian due to the choice (8) for . But because the wavefunction and its first derivative are continuous across the channel surface, the projection $(c|\Psi)$ is simply Eq. (1) evaluated at $r_c = a_c$. Thus, Eq. (10) leads to a relation between elements of the R matrix,

$$R_{c'c} \equiv (c'|G|c) \quad , \tag{12}$$

and elements of the unitary collision matrix $U_{c'c}$ appearing in Eq. (1). In matrix form, this relation is [3]

$$U = 2iO^{-1}[1-R(L-B)]^{-1}RO^{-1} + IO^{-1} \quad , \tag{13}$$

where the incoming and outgoing spherical waves are evaluated at $r_c = a_c$, as is the logarithmic derivative $L_c = a_c \frac{\partial}{\partial r_c} O_c/O_c$.

The unitarity of the collision matrix U follows from the hermiticity of the R matrix. Furthermore, R, being surface matrix elements of the internal Green's function (11) depends only upon the properties of the internal hamiltonian H, which is dominated by nuclear forces. External Coulomb and angular momentum effects are separated out in Eq. (13) and contained in the surface functions O, I, and L. The real and imaginary parts of $L = S + iP$ are usually called the "shift" and "penetrability" functions, respectively, while the phase of O is termed the "hard sphere" phase shift, ϕ.

The hermiticity of G allows a simple and familiar expansion for the R matrix. Since the eigenfunctions $|\lambda)$ satisfying

$$(H +)|\lambda) = E_\lambda|\lambda) \quad , \tag{14}$$

for real eigenvalues E_λ form a complete orthonormal set in the internal region, G has the spectral expansion

$$G = \sum_\lambda \frac{|\lambda)(\lambda|}{E_\lambda - E} \quad , \tag{15}$$

from which it follows immediately that

$$R_{c'c} = (c'|G|c) = \sum_\lambda \frac{\gamma_{c'\lambda}\gamma_{c\lambda}}{E_\lambda - E} \quad , \tag{16}$$

where $\gamma_{c\lambda} = (c|\lambda)$ is the "reduced width" amplitude.

Equations (13) and (16) constitute the simple unitary, symmetric, multichannel description of nuclear reactions desired for parametric fits to experimental data. The parameters of R-matrix theory, the $\gamma_{c\lambda}$ and E_λ, depend in principle upon the channel radii a_c and boundary conditions B_c. But since one can transform the parameters $\gamma_{c\lambda}, E_\lambda$ analytically from one boundary condition to another [8], the values of B_c are of no practical consequence (although they can be important in interpretations of the $\gamma_{c\lambda}$ and E_λ). In principle, the same is true of the channel radii a_c, provided that they always exceed the range of the short-ranged inter-cluster forces. But in practice, the sum over levels in (16) is always truncated, and the correspondence of the $\gamma_{c\lambda}$ and E_λ for different radii is difficult to establish. Fortunately, it appears in most cases that truncated level expansions can give good descriptions of the data over finite energy regions if the channel radii are close to the sizes of the interacting nuclei [$a_c \approx r_0(A_1^{1/3} + A_2^{1/3})$].

The simple pole terms of the R-matrix expansion can be made to correspond with resonances of the interacting system of particles. However, distant-level, or "background" (sometimes called R_∞) poles usually identified with shorter-lived "direct" processes can also be included. Both types of terms are important in describing reactions in light nuclei, and the off-diagonal distant-level contributions to the R matrix usually cannot be neglected since they correspond to direct stripping and pick-up mechanisms.

R-MATRIX CODES AND METHODS

Several codes have been developed to fit experimental data with R-matrix parameters. The emphasis of some of the work done with these codes is on nuclear structure studies, where the main interest is in the values of the parameters; and of others, evaluation, where the fits to the data are of primary concern. In either case, the calculational procedures are much the same. One choses for a set of two-body channels in the system of interst (usually the open channels in the energy range under consideration) values of channel radii and maximum orbital angular momentum quantum numbers (ℓ_{max}). This defines a finite number of states for the problem, coupled according to their values of total angular momentum and parity (J^P). For each J^P, a finite number of levels is specified by choosing level eigenenergies E_λ and channel reduced widths $\gamma_{c\lambda}$. Starting values for many of these parameters can be obtained from compilations of nuclear structure data [9] or from theoretical calculations. The R-matrix elements are formed according to Eq. (16) and combined with the surface quantities derived from Coulomb wavefunctions to give the collision matrix elements of Eq. (13), which are then used in Eqs. (3)-(5) (or in corresponding relations) to calculate the cross sections (or other observables). The R-matrix

parameters are adjusted to acheive a "best" fit in some sense to the experimental data included in the analysis.

Characteristics of some of the R-matrix codes currently being used are listed in Table I. MULTI [10] has been used in nuclear-structure studies for neutrons incident on a variety of heavy and light elements. Codes in the ORMAP sequence [11] have been used to analyze and extract nuclear-structure information from neutron elastic and inelastic measurements done on light targets between ^{6}Li and ^{13}C at Ohio University .

The RFUNC program is a single-channel (R-function) code used at ORNL to evaluate the n-^{12}C cross sections at energies below 2 MeV [12] for ENDF/B. A similar R-function program is used at Yale University and at ANL. EDA [13] has been used to analyze data for both neutron- and charged-praticle-induced reactions in light systems between A = 4 and A = 17. The ENDF/B evaluation for ^{4}He, and those for ^{6}Li, ^{10}B, ^{15}N, and ^{16}O at low energies, were based on R-matrix analyses using this code. EDA is the most general R-matrix program available, but it requires large computers, whereas most of the other codes can be run on a PDP-10.

Different methods of preparing the input data are used in analyses done with these codes. MULTI and EDA use the data directly, weighted in most cases by the quoted experimental errors, assuming no correlations (other than overall normalizations) exist among the measurements. ORMAP analyzes Legendre coefficients derived from the experimental data, neglecting correlations among the input coefficients. In their ^{12}C evaluation, Fu and Perey [12] first estimated averaged values and covariances for six individual sets of total cross-section measurements, then combined them using Baye's theorem to obtain a significantly reduced number of points and associated covariances for input to their R-matrix analysis.

Three of the codes (MULTI, RFUNC, and EDA) use automated search routines to minimize the χ^2 of their fits to the input data, thereby obtaining information about the second derivatives of χ^2 with respect to the R-matrix parameters. Twice the inverse of the matrix of these second derivatives evaluated at the parameter values which minimize the χ^2 gives the covariance matrix C for the parameters p. A straightforward application of first-order error propagation gives for the covariances among the calculated cross sections,

$$\mathrm{cov}\,(\sigma_i, \sigma_j) = \sum_{k\ell} \frac{\partial \sigma_i}{\partial p_k} \, C_{k\ell} \, \frac{\partial \sigma_j}{\partial p_\ell} \quad , \tag{17}$$

where σ_i and σ_j can be cross sections for difference reactions, for different energies, and for different angles, and the derivatives $d\partial/d\partial$ are evaluated at the parameter values which minimize χ^2. Such calculations were used to provide the covariance files at low energies for the ENDF/B evaluations of ^{6}Li, ^{10}B, and ^{12}C.

EXAMPLES

In this section, we present specific examples from analyses done with EDA to enlarge on some of the points made in the introductory sections and to illustrate R-matrix techniques which appear to account rather successfully for a large body of data from reactions among light nuclei.

Low-Energy Behavior of Cross Sections

At low energies, one expects the incident particle to be affected mainly by the long-ranged parts of the interaction due to its long wavelength. These are the parts of the interaction (i.e., Coulomb and angular momentum effects) that are treated "exactly" in R-matrix theory. Indeed, all the simple dependencies one expects for low-energy cross sections--"1/v" for exoergic neutron-induced reactions, "constant" for neutron elastic scattering with hard-sphere s-waves, "Gamow" for charged-particle induced reactions, etc.--come automatically from the penetrability functions P of the theory. These simple dependencies are modified by the short-ranged effects contained in the R-matrix itself, which can be significant in the case of low-lying resonances.

As a first example, we show in Fig. 1 the ratio of the $^{10}B(n,\alpha)$ and $^{6}Li(n,\alpha)$ cross sections as calculaed from the R-matrix analyses of reactions in the ^{11}B and ^{7}Li systems to provide n-^{10}B and n-^{6}Li cross sections at low energies for ENDF/B-V, compared to recent measurements done at NBS. The measured and calculated shapes are normalized to 1 at 15 eV, so that one has only a comparison of the shapes of the cross sections as a function of energy. If both cross sections were strictly 1/v, the ratio would plot as a horizontal straight line, and in fact, one sees about a 2% deviation from this behavior up to 1 keV in both the measurements and the calculation. In the calculations, the deviation comes from a broad 7/2+ s-wave resonance which causes the $^{10}B(n,\alpha)$ cross section to drop below 1/v. The $^{6}Li(n,\alpha)$ cross section, having no low-lying s-wave resonances, remains close to 1/v in this region. The break in the measurements at energies below 10 eV is not yet understood, but is believed to be a molecular effect in the $^{10}BF_3$ crystal.

A similar plot in Fig. 2 shows the ratio of the $^{10}B(n,\alpha)$ to $^{3}He(n,p)$ cross sections at energies below 50 keV. Again, the data are recent measurements by the NBS group, the solid curve is the R-matrix calculation, and the dashed curve is the present ENDF/B evaluation. A significant departure from 1/v is caused in this case by a 0^+ resonance in the $^{3}He(n,p)$ cross section located just below the n-^{3}He threshold. The departure enters gradually in the R-matrix calculation, as the data indicate, whereas the evaluation, without the guidance of low-energy data, was forced to make a rather abrupt transition between regions of 1/v and non-1/v behavior in the cross section.

Fig. 3 shows an example for an important charged-particle fusion process, the $T(d,n)^4He$ reaction. The Gamow penetrability and $1/k^2$ factors have been removed from the cross section, leaving an "astrophysical S-function" which presumably behaves as a constant at low energies. In fact, the dashed horizontal line labled "Gamow extrapolation" corresponds to the cross-section values reported by Arnold et al. [14] in place of their own experimental measurements [15] at energies below 20 keV. The R-matrix calculation clearly does not follow the Gamow dependence at low energies due to a $3/2^+$ s-wave resonance at 100 keV, and tends to confirm the behavior of the original measurements [15].

Unitary Constraints Near Resonances

As was mentioned in the introduction, unitary constraints near resonances can approach the simple situation illustrated by the single 2 x 2 collison matrix. A good example is the cross sections for $n-^6Li$ near the 245 keV resonance, which has two states coupled to $J^P = 5/2^-$. A quick look back at Eqs. (1), (4), and (5) will show that for δ_l resonant ($\approx \pi/2$), the peak total and reaction cross sections are both determined by the η parameter alone. Thus, the failure of the ENDF/B-IV R-matrix analysis to fit both Diment's [16] relatively precise total cross sections and measurements [17,18] of the $^6Li(n,t)$ cross section near the peak, as shown in Fig. 4, signaled a severe (>15%) unitary inconsistency between measurements of the neutron cross sections. Additional input on the values of η was gained from precise (2,2) $t-\alpha$ elastic scattering measurements [19] over the resonance, which indicated that the calculated peak total cross section needed to be raised and the peak (n,t) cross section somewhat lowered (the anti-correlation between the peak total and (n,t) cross sections is also a consequence of unitarity). Cross sections from the revised analysis (ENDF/B-V) including the precise $t-\alpha$ data are seen in Fig. 5 to agree well in the peak of the resonance with later measurements of the total cross section [20] and of the (n,t) cross sections [21,22], achieving unitary consistency among the cross sections for this system near the peak of the resonance to the order of 3%.

Charge-Symmetric Techniques

The fact that the R-matrix has the same symmetries as the internal hamiltonian, which is dominated by nuclear forces, can be exploited to introduce symmetry properties not shared by the collision matrix. One of these is the charge symmetry of nuclear forces, which means that nuclear forces in a system are unchanged by the interchange of protons and neutrons. This implies that the R-matrix parameters for mirror systems are essentially the same if the boundary condition numbers B_c are taken to be the same. The parameters need to be corrected for internal Coulomb effects which can be treated perturbatively, as was described at the Harwell meeting [23].

We have used charge-symmetric R-matrix analyses to describe successfully data from mirror reactions in the 4-, 5-, and 7-nucleon systems. The ENDF/B ^{4}He evaluation, for instance, comes from an R-matrix analysis which describes n-α and p-α data simultaneously with essentially the same parameters. The example shown in Fig. 6 is a charge-symmetric prediction for the n-T total cross section from an R-matrix analysis of p-^{3}He scattering data below 20 MeV. The differences between the prediction (solid curve) and the ENDF/B evaluation (dashed curve), which is representative of all but the most recent data, are quite large at energies below 1 MeV. However, a precise new measurement of the total cross section done at LLL [24] appears to confirm the prediction at low energies, which resolves a long-standing conflict between measurements of the low-energy n-T cross section and those of the coherent scattering length [25].

LIMITATIONS OF CONVENTIONAL R-MATRIX THEORY

The first limitation one encounters in applying R-matrix theory to the description of light nuclear reactions is the restriction to two-body channels. Especially when deuterons are involved, the thresholds for three- (and more) body channels occur at relatively low energies. Such channels can be accounted for approximately by treating them as pseudo two-body channels in which pairs of the particles are resonant. This treatment is usually adequate to account for absorption in the two-body channels due to three-body states and can even be used to fit three-body spectra if the widths of the resonating sub-structures are properly taken into account.

The second limitation of the theory that is important in applications to light nuclei is the assumption that the channels for different arrangements are orthogonal at finite channel radii. This orthogonality is automatic for channels having the same two-body arrangement, and holds for channels from different arrangements if the channel radii are infinite, but in general the overlap is non-zero for channels from different arrangements at finite channel radii.

Including channel non-orthogonality affects the relation (13) between the R matrix and the collision matrix, U, in that the quantities I and O no longer remain simple, diagonal matrices of Coulomb wavefunctions. Unfortunately, the off-diagonal elements of these matrices become complicated integrals that cannot be evaluated in a model-independent manner. However, taking such contributions into account with a simple three-particle model, for instance, would build into R-matrix theory the particle-exchange effects that have been speculated to be important in explaining, for example, the large 1/v cross section at low energies in the ^{6}Li(n,t)^{4}He reaction.

CONCLUSIONS

The introductory discussion and examples given were intended to present two main points; first, that unitarity is a necessary and useful theoretical constraint to impose upon the evaluation of microscopic nuclear data, and second, that R-matrix theory provides a unitary framework particularly well-suited for describing reactions in light nuclei. The full advantages of this approach for evaluation purposes are realized only by doing multi-reaction analyses, including data for a variety of observable types. This has been done, wherever possible, for the R-matrix-based evaluations for ^{4}He, ^{6}Li, ^{10}B, ^{12}C, ^{15}N, and ^{16}O, and perhaps should be used to update evaluations for some of the other light elements in ENDF/B.

Users of R-matrix codes for data evaluation appear to agree on the desirability of diluting the influence of any one data set on the final results, either by including a large number of data sets directly in the analysis or by precombining measurements of the same quantity. The second approach, if it is done in a statistically unbiased fashion, obviously has merit when some of the data sets contain unmanageably large numbers of points. The complication in this case, however, is that covariances must be supplied with the input data.

Finally, we conclude with the observation that this basically successful approach to describing charged-particle and neutron-induced reactions has some interesting areas for further study. A more sophisticated treatment of the internal Coulomb corrections we are making to R-matrix parameters would improve the predictive capability of this technique and possibly allow a better assessment of the consistency of data for mirror reactions with charge symmetry of nuclear forces. An approximate accounting for three-body channels and for channel non-orthogonality at finite radii could result in more detailed agreement with experimental measurements and a better undertanding of the origin of certain non-resonant features in the data.

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REFERENCES

1. G. Breit, Phys. Rev. 58, 1068 (1940).

2. E. P. Wigner and L. Eisenbud, Phys/ Rev. 72, 29 (1947).

3. A. M. Lane and R. G. Thomas, Rev. Mod. Phys. 30, 257 (1958).

4. H. P. Stapp, T. J. Ypsilantis, and M. Metropolis, Phys. Rev. 105, 302 (1957).

5. G. M. Hale, "R-Matrix Methods for Light Systems, IAEA-90, Vol. II, p. 1 (1976).

6. G. M. Hale and D. C. Dodder, "R-Matrix Analyses of Light-Elment Reactions for Fusion Applications." Proc. Int. Conf. on Nuclear Cross Sections for Technology, Knoxville (1979), to be published, and LA-UR-79-2896 (2979).

7. A. M. Lane and D. Robson, Phys. Rev. 51, 774 (1966).

8. F. C. Barker, Aust. J. Phys. 35, 341 (1972).

9. F. Ajzenberg-Selove, "Energy Levels of Light Nuclei," Nucl. Phys. A320 (1979), A336 (1980).

10. G. F. Auchampaugh, "MULTI, A FORTRAN Code for Least Squares Fitting of Neutron Cross Section Data Using the Reich-More Multilevel Formalism," LA-5473-MS (1974).

11. R. Lane and collaborators, ORMAP (Ohio R-Matrix Program), uppublished.

12. C. Y. Fu and F. G. Perey, Atom. Data and Nucl. Data Tables 22, 249 (1978).

13. D. C. Dodder, K. Witte, and G. M. Hale, EDA (LASL Energy Dependent Analysis Code), unpublished.

14. W. R. Anrold et a;/, Phys. Rev. 9, 483 (1954)

15. W. R. Arnold et al., "Absolute Cross Section for the Reaction T(d,n)He^4 from 10 to 120 keV," LA-1479 (1953).

16. K. M. Diment and C. A. Uttley, "The Total Cross Section of ^6Li," AERE-PR/NP 15, 12 (1969).

17. M. S. Coates, G. S. Hunt, and C. A. Uttley, "Measurements of the Relative $^6Li(n,\alpha)$ Cross Sections in the Energy Range 1 keV to 7500 keV," Neutron Standards Reference Data, IAEA, Vienna, p. 105 (1974).

18. W. P. Poenitz, Z. Phys. 268, 359 (1974).

19. R. A. Hardekofp et al., "$^4He(t,t)^4He$ Elastic Scattering: Analyzing Powers and Differential Cross Sections," LA-6188 (1977).

20. P. Guenther, A. Smith, and J. Whalen, "Neutron Total and Scattering Cross Sections of ^6Li in the Few MeV Region," ANL/NDM-52 (1980), and personal communication from A. Smith (1980).

21. G. P. Lamaze, R. A. Schrack, and O. A. Wasson, Nucl. Sci. Eng. 68, 183 (1978).

22. C. Renner et al., Bull. Am. Phys. Soc. 23, 526 (1978).

23. D. C. Dodder and G. M. Hale, "Applications of Approximate Isospin Conservation in R-Matrix Analyses," in Neutron Physics and Nuclear Data, Proceedings (Harwell), p. 490 (1978).

24. T. W. Phillips, B. L. Berman, and J. D. Seagrave, Phys. Rev. C22, 384 (1980).

25. H. Rauch, "The Low Energy Neutron Scattering Lengths of ^{3}He and T and Their Relation to the Four-Body Problem," in Few Body Systems and Nuclear Forces I, (Proceedings, Graz), p. 289 (1978).

TABLE I

CHARACTERISTICS OF SOME R-MATRIX CODES IN CURRENT USE

Code:	MULTI	ORMAP	RFUNC	EDA
Developed at:	LLL(LASL)	Ohio U	ORNL	LASL
Incident particles:	neutrons	neutrons	neutrons	general
No. arrangements:	3	3	1 (spin 0 target)	no limit*
No. levels/J^P:	no limit*	5	10	no limit*
ℓ max:	6	3	3	no limit*
Types of data analyzed:	Int. cross sections for (n,n), (n,n'), (n,γ), (n,f)	Legendre coeffi-cients for σ,P up thru L=5	σ,P	general
Search Method:	Automated Levenberg Marquardt algorithm	Manual	Automated grid	Automated rank-1 variable metric
Fits on:	PDP-10	PDP 10	PDP-10	CDC-7600

* Subject to overall storage limitations

FIGURE CAPTIONS

1. Ratio of relative shapes of the $^{10}B(n,\alpha)$ cross sections at energies between 1 eV and 1 keV. The points are recent measurements from NBS, and the curve if from R-matix calculations used in the ENDF/B-V evaluations for ^{10}B and ^{6}Li.

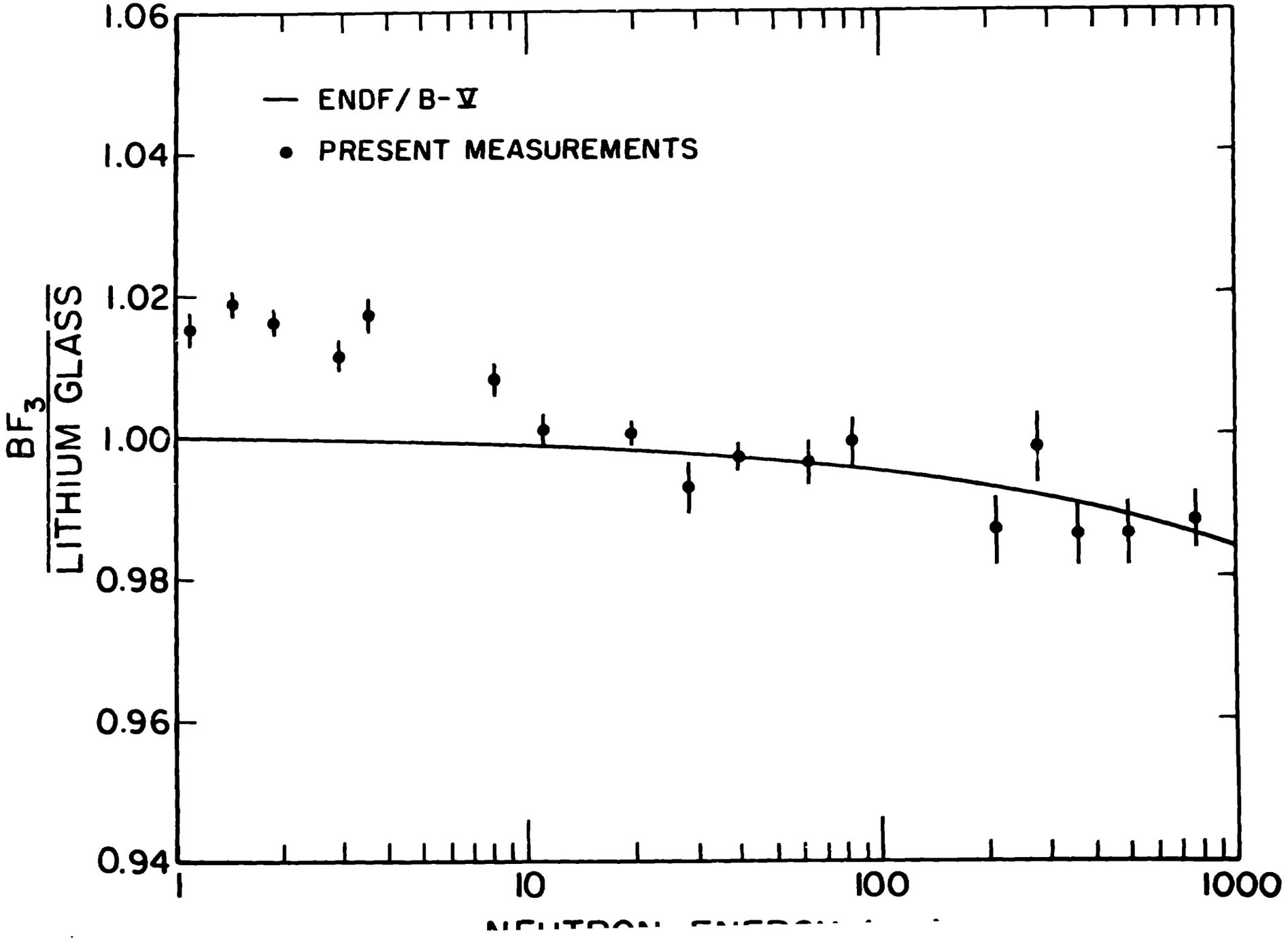
2. Ratio of the relative shapes of the $^{10}B(n,\alpha)$ and $^{3}He(n,p)$ cross sections at energies between 10 eV and 50 keV. All the points are measurements by the NBS group. The solid curve is a claculation of ENDF/B-V $^{10}B(n,\alpha)$ cross section relative to the $^{3}He(n,p)$ cross section obtained from an R-matrix analysis of reactions in the ^{4}He system; the dashed curve is for the same $^{10}B(n,\alpha)$ cross section relative to the ENDF/B-V $^{3}He(n,p)$ cross sections.

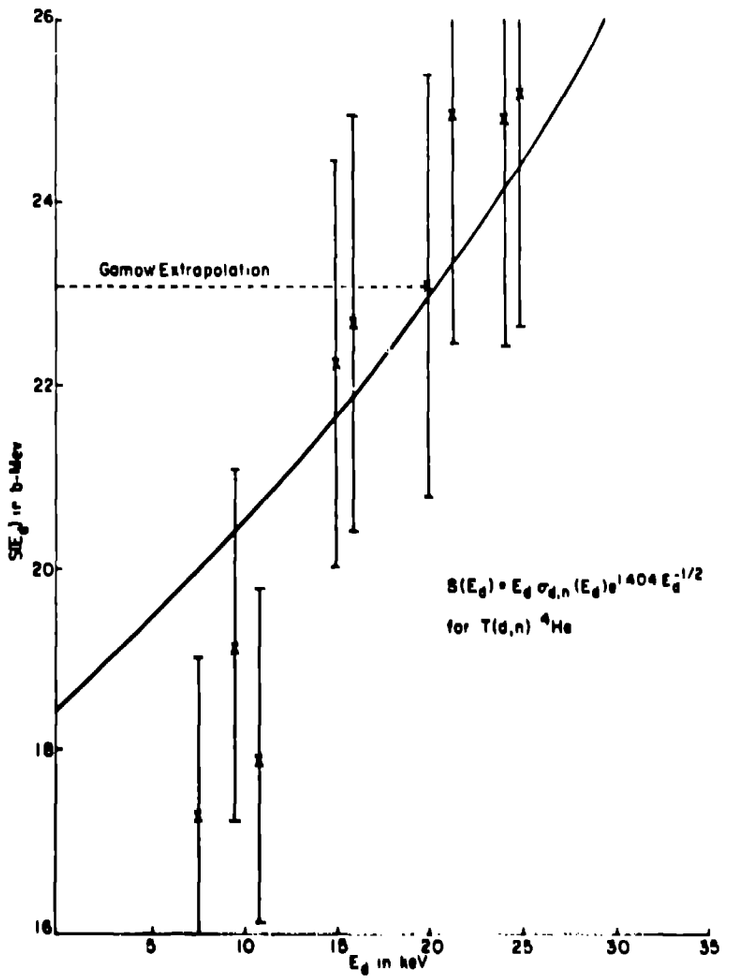
3. Astro physical S-function for T(d,n)^{4}He as a function of laboratory energy. The solid curve is the R-matrix calculation, and the dashed line is the Gamow extrapolation reported by Arnold et al. [14] in place of their measured points [15] at energies below 20 keV.

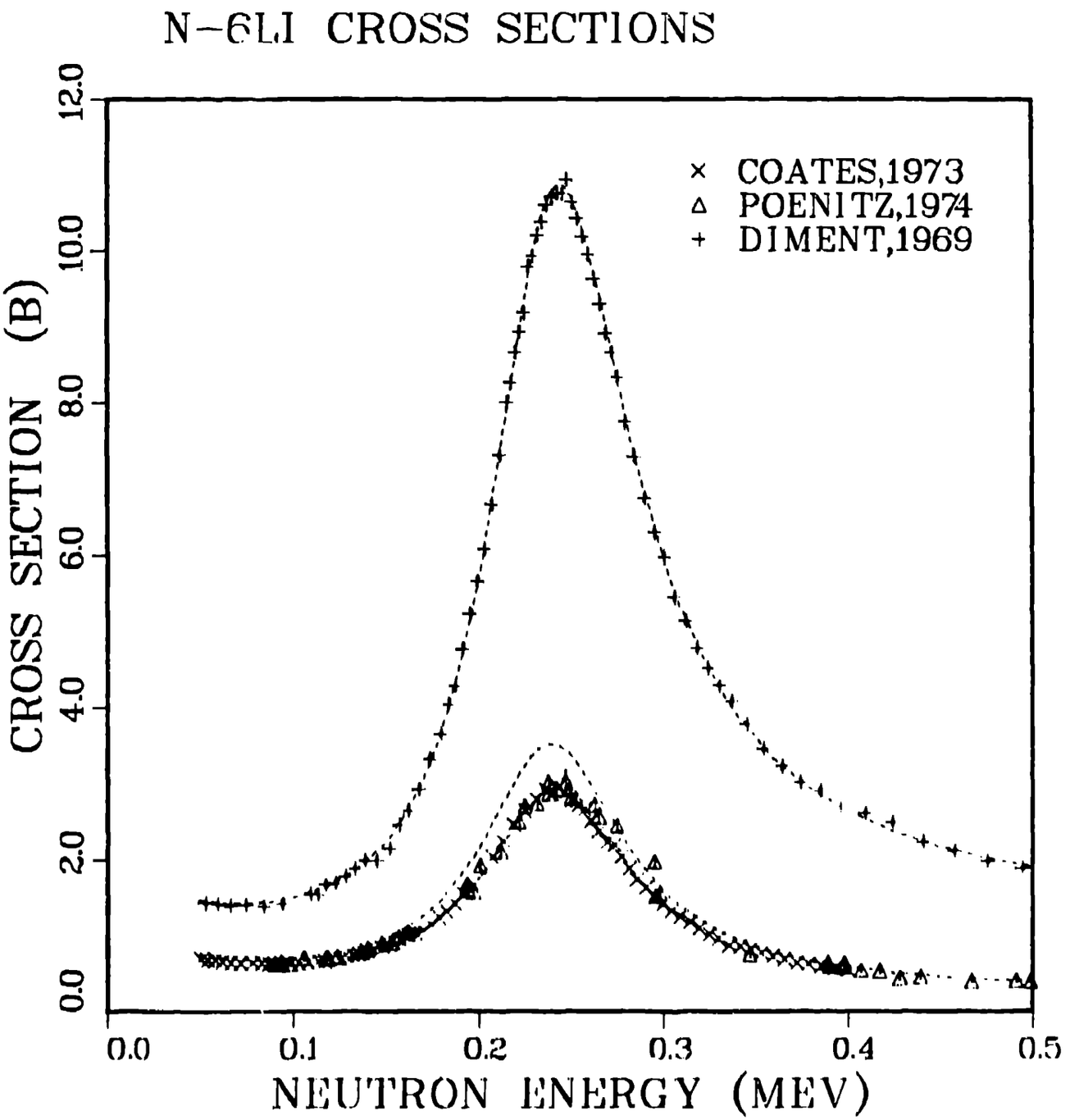
4. Measurements of the n-^{6}Li total cross section by Diment [16] and of the ^{6}Li(n,t) reaction cross section by Coates [17] and by Poenitz [18] compared to calculated values (solid curves) from the R-matrix-based ENDF/B-IV evaluation.

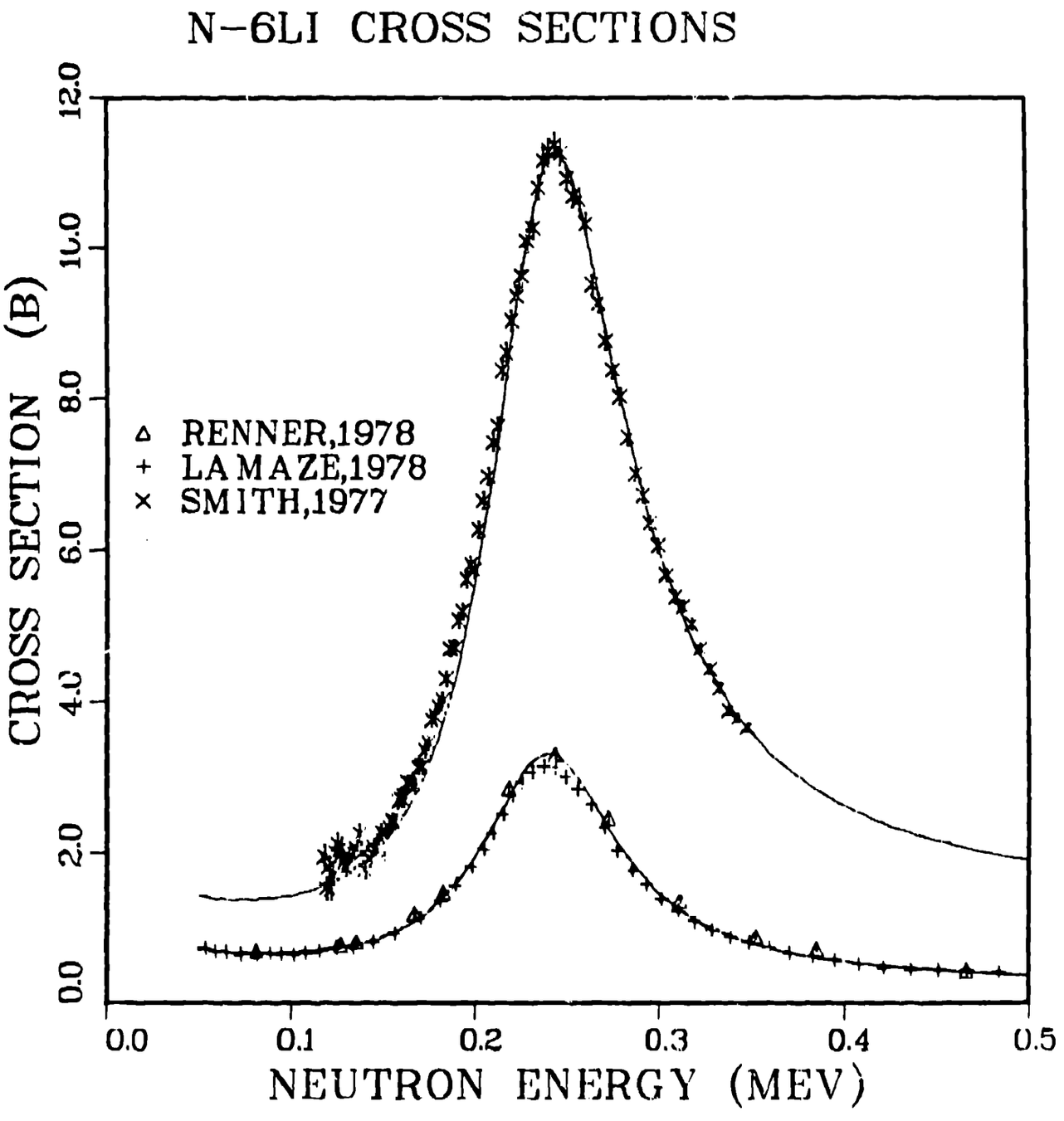
5. Recent measurements of the n-^{6}Li total cross section by Smith [20] and of the ^{6}Li(n,t) reaction cross section by Lamaze [21] and by Renner [22] compared to calculated values (solid curves) from the R-matrix-based ENDF/B-V evaluation.

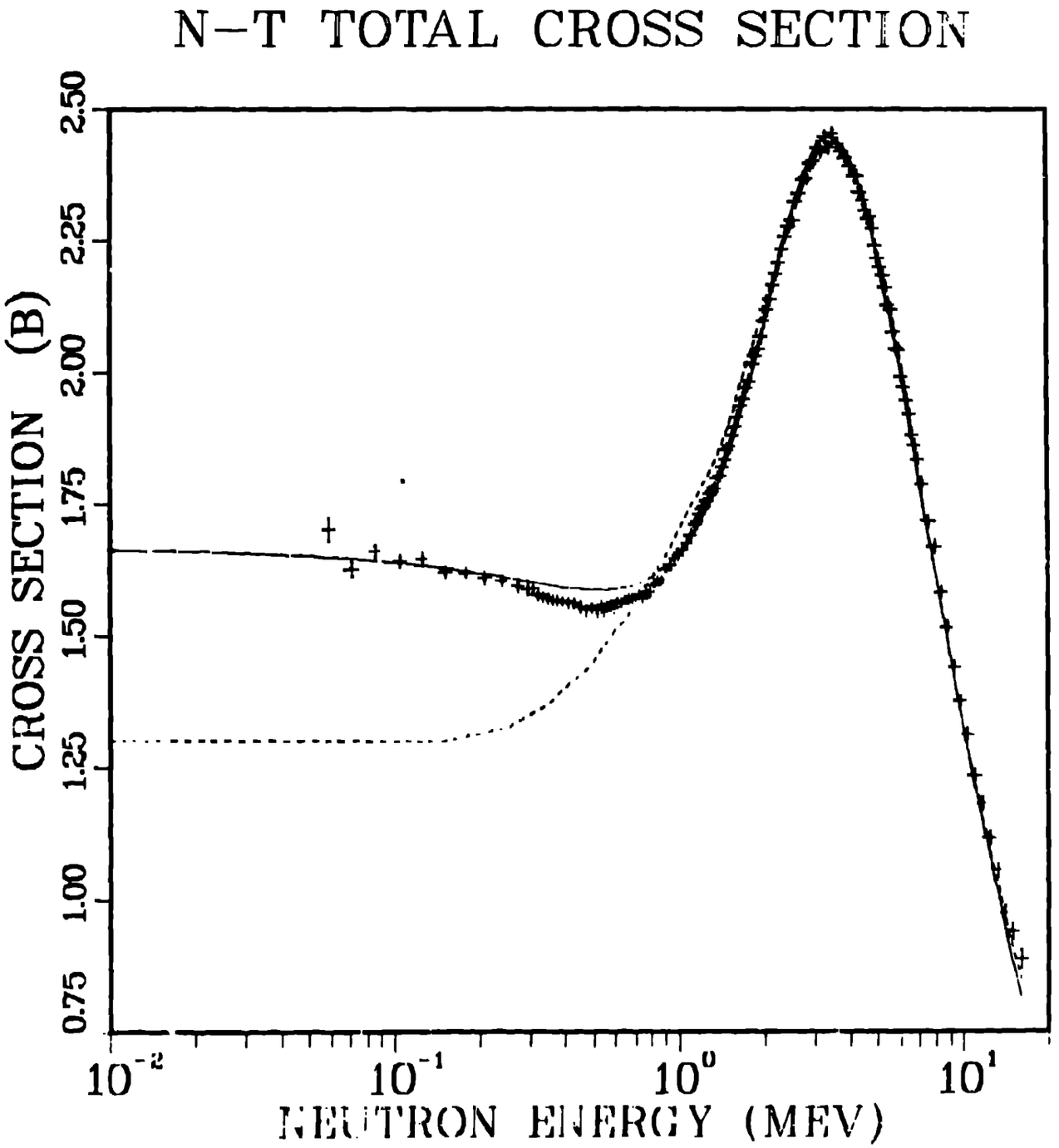
6. Coulomb-corrected, charge-symmetric prediction of the n-T total cross section (solid curve) from an R-matrix analysis of p-^{3}He scattering, compared with recent measurements by Phillips [24] and with the ENDF/V-V evaluation (dashed curve), which is representative of all previous measurements.


1.06
1.04
1.02
1.00
0.98
0.96
0.94
— ENDF/B-V
• PRESENT MEASUREMENTS
BF3 / LITHIUM GLASS
1
10
100
1000


26
24
22
20
18
16
Gamow Extrapolation
S(E_d) in b-Mev
S(E_d) = E_d σ_d,n (E_d) e^1.404 E_d^-1/2
for T(d,n) ⁴He
5
10
15
20
25
30
35
E_d in keV


N-6LI CROSS SECTIONS
CROSS SECTION (B)
NEUTRON ENERGY (MEV)
× COATES,1973
△ POENITZ,1974
+ DIMENT,1969
0.0
2.0
4.0
6.0
8.0
10.0
12.0
0.0
0.1
0.2
0.3
0.4
0.5


N-6LI CROSS SECTIONS
CROSS SECTION (B)
NEUTRON ENERGY (MEV)
RENNER,1978
LAMAZE,1978
SMITH,1977
0.0
2.0
4.0
6.0
8.0
10.0
12.0
0.0
0.1
0.2
0.3
0.4
0.5


N-T TOTAL CROSS SECTION
CROSS SECTION (B)
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
10^{-2}
10^{-1}
10^{0}
10^{1}
NEUTRON ENERGY (MEV)